Bleaching Wave Phenomena

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The propagation of a coherent beam in an absorbing medium is described as an energy transport process in which all the transporting particles are moving with the same direction and energy. In many problems involving a stationary medium, the coherent beam is the only significant energy flux. In this case the temperature is governed by a second-order partial differential equation called the beam equation. If the medium is bleachable (the transport proceeds more freely as the temperature increases), then the temperature profiles exhibit a wavelike propagation. In general, the beam equation can be integrated once leaving a first-order equation. Three known explicit solutions to the first-order equation are presented: laser heating of a stationary plasma; laser irradiation of a two level substance; and the laser induced breakdown wave.

A CONSEQUENCE of the second law of thermodynamics is that any natural process tends to smooth macroscopic nonuniformities. The leveling of the nonuniformity may occur by either macroscopic or microscopic motion, or both. If macroscopic motion dominates, then a dynamical process ensues. If microscopic motion dominates, then a transport process ensues.

In recent years the development of powerful energy sources such as lasers and electron beams has made it possible to create media having very high temperatures and large temperature gradients. If the energy deposition is rapid, the early stages may be dominated by a transport process (such as thermal conduction) in a stationary medium. In such cases, hydrodynamic motion becomes important only at later times and in fluid media.

A transport process may exhibit somewhat different forms involving separate mathematical descriptions. If the mean free path of the transporting particles is small compared to the characteristic distance in which temperature changes significantly, then one sees diffusive transport which is governed by the diffusion equation. However, if the mean free path is comparable to or large compared to the characteristic distance, then one has nondiffusive transport and the mathematical description is not at all obvious. In the extreme case however, the mathematics is very simply described by what may be called the beam equation. This is the limit in which all the transporting particles have the same energy and direction, i.e., a coherent beam.

It is the mathematics of coherent transport which is examined in detail in this paper. Of special interest is the case where energy transport proceeds more freely as the temperature increases, leading to bleaching wave phenomena. This kind of phenomenon can appear for both diffusive and coherent transport.

Governing Equations

Neglecting hydrodynamic motion and energy source terms, the conservation of energy requires that the rate of increase of material internal energy be equal to the divergence of the transported energy flux.

$$\rho C_v(\partial T/\partial t) = -\nabla \cdot \bar{S} \tag{1}$$

where $\rho = \rho(x)$ is the material density; C_v is the constant volume specific heat (henceforth assumed constant); $T = T(x, t)$ is the temperature; and $\bar{S}$ is the transported energy flux vector. The energy flux in general includes such things as thermal conduction due to collisions between particles or otherwise, and radiant energy transport due to photon emission or a coherent beam. The assumption of no motion restricts one to short time scales in the case of fluids. The assumption of no source terms disallows cases where there is some but not complete equilibration between multiple species, such as electrons and ions in plasmas. An analysis of regimes where equilibration, and motion are not important is given elsewhere for the particular example of a laser heated plasma.[1]

If the dominant flux is diffusive then an equation of state can be added which is commonly called the Fourier heat conduction law, $\bar{S} = -K\nabla T$, where K is the conduction coefficient. This conduction may represent collisional thermal conduction or the diffusion of thermal radiation as discussed by Zel'dovich and Raiser.[2] Combining Eq. (1) and the Fourier law yields the nonlinear diffusion equation.

$$\rho C_v(\partial T/\partial t) - \nabla \cdot (K\nabla T) = 0 \tag{2}$$

The behavior of Eq. (2) is well understood and is not given detailed consideration in this paper. A review of the mathematics of nonlinear diffusion was given by Crank[3] and a numerical approach to its solution was discussed by Richtmeyer.[4]

The problem to be examined in this paper, however, is coherent transport. If the dominant flux is a coherent beam, then the energy flux is given in terms of the beam intensity $\bar{S} = I\delta(\Omega - \Omega_o)\bar{\Omega}_o$ where $\bar{\Omega}_o$ is a unit vector in the direction of the beam, $\delta(\ )$ is the delta function, and I is the intensity of the beam. If the beam is moving in the $+x$ direction, then the divergence term in Eq. (1) is given by $\nabla \cdot \bar{S} = \partial I/\partial x$. The intensity is governed by a radiative transfer equation.

$$\partial I/\partial x + I/\ell = 0 \tag{3}$$

where $\ell = \ell(\rho, T)$ is the absorption length of the beam. It is assumed that scattering is negligible compared to absorption. Note also that the retardation term (due to the finite value of the speed of light) has been neglected. This merely assumes that the speed of light is much greater than any other velocities that may appear in the problem. Then Eqs. (1) and (3) combine to eliminate intensity and yield a second-order partial differential equation, which might be called the beam equation

$$\partial[\ell\partial(\rho T)/\partial t]/\partial x + \partial(\rho T)/\partial t = 0 \tag{4}$$

Recall that Eq. (4) assumes only coherent energy transport. This eliminates thermal conduction and spontaneous emission, which is valid for sufficiently short time scales.

The initial condition is the spatial temperature distribution at time $t = 0$, $T(x, O) = T_o(x)$. The basic boundary condition is that the intensity at the edge of the substance ($x = 0$) is equal to the intensity delivered by the source minus that reflected at the boundary ($x = 0$). Thus $I(O, t) = I_o(t)$. This boundary condition

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can be written using temperature as the dependent variable by applying Eqs. (1) and (3).

$$I_o(t) = [C_v \ell\, \partial(\rho T)/\partial t]_{x=0} \tag{5}$$

The second-order partial differential equation, Eq. (4); the initial condition; and the boundary condition, Eq. (5) form a well-posed problem.

By a simple manipulation of the variables, it is possible to integrate the beam equation with respect to time leaving only a first-order equation to be solved. The absorption length can be alternatively expressed as a function of ρ, and ρT: $\ell = \ell(\rho, T) = \ell^*(\rho, \rho T)$. It is convenient to introduce $L(\rho, \rho T)$, the antiderivative of ℓ with respect to the variable ρT so that $\partial L/\partial(\rho T) = \ell^*$. If L exists and has continuous second partial derivatives, then Eq. (3) reduces to

$$\partial(\partial L/\partial x + \rho T)/\partial t = 0 \tag{6}$$

Using the initial condition $T(x, O) = T_o(x)$, Eq. (6) can be integrated with respect to time to yield,

$$\partial[L(\rho, \rho T) - L(\rho, \rho T_o)]/\partial t + \rho T - \rho T_o = 0 \tag{7}$$

The boundary condition can likewise be integrated with respect to time giving

$$[L(\rho, \rho T) - L(\rho, \rho T_o)]_{x=0} = \frac{1}{C_v}\int_0^t I_o\, dt' \tag{8}$$

Thus the problem reduces to a first-order partial differential equation in which only derivatives with respect to x are seen to appear.

If the absorption length happens to be constant, then L is just $\rho T \ell^*$. Then the familiar exponential decay temperature profiles will appear. For example, if the initial temperature is zero, density, beam intensity, and absorption length are constant, then the temperature profiles are

$$T = (I_o/\rho C_v \ell^*) t e^{-x/\ell^*}$$

The temperature is exponentially damped as one moves into the plasma, and rises with a uniform time factor as time moves on. This illustrates the basic character of linear absorption.

Bleachable Media

In most physical problems the absorption length depends on temperature, and often the dependence is very strong. When powerful beams are in view, it is not usually valid to assume constant temperature. Hence linear absorption is no longer adequate to describe the phenomenon.

In a large number of cases, the absorption length increases with temperature. Such a material irradiated by a powerful beam exhibits an unusual behavior. If the material is initially cold, then its absorption length may be quite short. Hence the beam is absorbed in a fairly thin layer at the edge of the substance. The strong absorption in this layer raises its temperature and absorption length as a result, and quickly renders it transparent to the beam. Then a deeper layer is exposed to nearly the full power of the beam, is heated and in turn is rendered transparent.

As successive layers of the substance are heated, and rendered transparent, the beam bores its way into the plasma in a wavelike fashion. The temperature profiles exhibit the same character. Such behavior may be called a bleaching wave and is to be expected any time a powerful beam is applied to a bleachable medium, i.e., one in which the energy transport proceeds more freely as the temperature is raised. This is equivalent to saying that absorption length increases with increasing temperature.

The bleaching wave in the coherent energy transport problem has an analog in the case of diffusive energy transport, which is governed by Eq. (2). A medium for which the conduction coefficient increases with increasing temperature is also bleachable in that energy transport proceeds more freely as the temperature is raised. Thus energy added at a point or plane in the medium will initially only diffuse to the adjacent layers due to the low value of the conduction coefficient. But as those layers are heated, the energy freely diffuses through them to the next layers and so on. The result is the thermal wave[2] which is related to the bleaching wave. In some problems, absorption length decreases with increasing temperature. In these cases, the medium becomes more and more resistant to energy transport as the temperature increases. Hence no bleaching wave appears.

The governing equations, Eqs. (7) and (8) apply to both kinds of media. They cannot be integrated analytically in general but simple solutions have been found for three interesting problems leading to a wavelike behavior. These are laser heating of a stationary plasma, laser irradiation of a resonant two level substance, and the laser induced breakdown wave. The former two involve bleachable media and lead to the bleaching wave. The latter does not involve a bleachable medium but leads to wavelike behavior for other reasons. Of course, each solution assumes negligible motion, emission, and thermal conduction during the laser pulse.

Laser Heating of a Stationary Plasma

Laser heating of a one-dimensional uniform stationary plasma with negligible thermal conduction and spontaneous emission was first studied by Rehm.[5] Later, the solution was found for an arbitrary density profile by Steinhauer and Ahlstrom.[1] The absorption length taking into account stimulated emission is[6]

$$\ell(\rho, T) = \ell_o T^{3/2}(1 - \rho/\rho_c)^{1/2}/\rho^2 \tag{9}$$

where ℓ_o and ρ_c (the critical density) are constants depending on the laser frequency. Eq. (9) is valid in the limit $h\nu \ll kT$ where h is Planck's constant, k is the Boltzmann constant, and ν is the laser frequency.

The antiderivative of $\ell^*(\rho, \rho T)$ with respect to ρT is

$$L = \tfrac{2}{5}\ell_o(\rho T)^{5/2}(1 - \rho/\rho_c)^{1/2}/\rho^{7/2} \tag{10}$$

Applying Eq. (10) to Eq. (7) yields the governing differential equation

$$[2\ell_o(1 - \rho/\rho_c)^{1/2}/5\rho^{7/2}](\partial/\partial x)[(\rho T)^{5/2} - (\rho T_o)^{5/2}] + \rho T - \rho T_o = 0 \tag{11}$$

where $T_o = T_o(x)$ is the initial condition. Applying Eq. (10) to Eq. (8) gives the boundary condition.

$$\left\{\frac{2\ell_o(1 - \rho/\rho_c)^{1/2}}{5\rho^{7/2}}[(\rho T)^{5/2} - (\rho T_o)^{5/2}]\right\}_{x=0} = \frac{1}{C_v}\int_0^t I_o\, dt' \tag{12}$$

For strong laser pulses, the initial temperature of the plasma is quite small compared to the final temperature. For simplicity in this example, the initial temperature is taken to be zero, $T_o(x) = 0$. Taking into account the small initial temperature merely adds a small correction to the solution.[1]

For zero initial temperature, Eq. (11) can be manipulated to yield a simple linear first-order differential equation for $T^{3/2}$

$$\frac{\partial T^{3/2}}{\partial x} - \frac{3}{5}\frac{1 - \rho/2\rho_c}{\rho(1 - \rho/\rho_c)}\frac{d\rho}{dx}T^{3/2} = -\frac{3}{2}\frac{\rho^2}{\ell_o(1 - \rho/\rho_c)^{1/2}} \tag{13}$$

This is a partial differential equation, but can be solved by the techniques for ordinary differential equations since only the derivative with respect to x appears. Subject to the boundary condition, the solution is

$$T = \frac{(\rho_c\rho/\ell_o)^{2/5}}{(1 - \rho/\rho_c)^{1/5}}\left[\left(\frac{5}{2\rho_c C_v}\int_0^t I_o\, dt'\right)^{3/5} - \frac{3}{2\ell_o^{2/5}\rho_c^{3/5}}\int_0^x \frac{\rho^{7/5}\, dx'}{(1 - \rho/\rho_c)^{1/5}}\right]^{2/3} \tag{14}$$

When the term in brackets becomes negative, then Eq. (14) fails. This corresponds to the unheated portion of the plasma where $T = 0$.

Examination of Eq. (14) clearly reveals the bleaching wave. The front of the wave is at the point where the term in brackets vanishes. Hence the velocity of the front can be calculated by an implicit differentiation with respect to time.

$$V = \frac{\ell_o^{2/5} I_o(t)(1 - \rho/\rho_c)^{1/5}}{C_v^{3/5}} \left(\frac{5}{2}\int_0^t I_o\, dt'\right)^{-2/5}$$

The region behind the front corresponds to a positive value of the term in brackets in Eq. (14) and hence is heated. The region ahead of the front is unheated. If the initial temperature of the plasma is relatively low but nonzero, then the region ahead of the front will be only weakly heated.

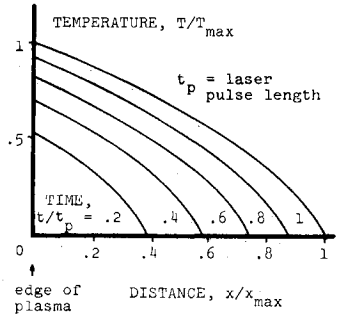

Fig. 1 Laser irradiation of a uniform plasma.

Temperature profiles for a special case have been calculated and are presented in Fig. 1. The plasma is assumed to be of uniform density. The laser intensity is assumed to be constant throughout the duration of the pulse, which arrives at the boundary $x = 0$ at time $t = 0$.

Laser Irradiation of a Two Level Substance

Suppose a substance irradiated with a laser pulse is composed of atoms having an allowed transition that is resonant with the laser frequency. If, as is usual, there are fewer atoms in the upper energy level than in the lower, photons from the laser will be absorbed. If, on the other hand, a population inversion exists, the laser pulse will be amplified as it passes through the substance. Practical examples of these phenomena are two level saturable absorbers and laser amplifiers.

The equations which apply to these two problems are the same and solutions to this problem were first worked out independently by Bellman et al.,[7] by Frantz and Nodvik,[8] and by Schulz-Dubois.[9] Each worked out the general time dependent solution in a two level substance. These papers neglected spontaneous emission and thermal relaxation from the energy levels, which is appropriate for short time scales. Bellman et al. pointed out the appearance of a phenomenon whereby the radiation bores its way through the substance with a certain velocity. This is simply a bleaching wave.

Presented below is a simple solution of laser irradiation of a two level substance. This calculation is simpler than that of the writers mentioned previously who took into account the finite speed of light in the radiative transfer equation.

The radiative transfer equation has the usual form, Eq. (3). For this problem the energy conservation equation takes a slightly different form,

$$h\nu(\partial n_u/\partial t) = -\partial I/\partial x \qquad (15)$$

where n_u is the number density of particles in the upper state and is analogous to the temperature in Eq. (1). h is Planck's constant and ν is the frequency of the radiation. Of course the sum of the number densities in the upper (n_u) and lower (n_ℓ) states is a constant (n).

In a two level substance, the absorption length is $\ell = 1/\sigma_\nu(n_\ell - n_u)$ where σ_ν is the cross section for an atom of the substance to absorb a photon of frequency ν. The initial condition is $n_u(x, O) = n_{uo}$. The basic boundary condition is $I(O, t) = I_o(t)$.

The antiderivative of ℓ is $L = -(\frac{1}{2}\sigma_\nu)\log(n - 2n_u)$. Thus for this problem, Eq. (11) becomes

$$-(1/2\sigma_\nu)(\partial/\partial x)[\log(n - 2n_u) - \log(n - 2n_{uo})] + n_u - n_{uo} = 0 \qquad (16)$$

Note that ρT in Eq. (11) has been replaced by n_u, and C_v by $h\nu$. The boundary condition becomes

$$-(h\nu/2\sigma_\nu)[\log(n - 2n_u) - \log(n - 2n_{uo})]_{x=0} = \int_0^t I_o\, dt' \qquad (17)$$

which is analogous to Eq. (12).

A particularly simple solution is available for the case n_{uo} = constant. Solving Eq. (16) for this case gives the solution,

$$\eta = \eta_0 + \tfrac{1}{2}(1 - \eta_0)\{1 + \tanh[(1 - \eta_0)(\xi_w - \xi)]\} \qquad (18)$$

where $\eta = 2n_u/n$ is the dimensionless measure of n_u. $\xi = x/\Delta x$, and $\tau = t/\Delta t$ are dimensionless distance and time, respectively. The length scale is $\Delta x = h_\nu/2\sigma_\nu \bar{I}_o$ and the time scale is $\Delta t = h\nu/2\sigma_\nu \bar{I}_o$, $\bar{I}_o$ being the time averaged incident intensity. The position of the center of the wave (where the surplus of particles $n_\ell - n_u$ is just half what it was initially) is given by

$$\xi_w = \frac{1}{2(1 - \eta_o)}\left\{\int_0^\tau \tilde{I}_o\, dt' + \log\left[1 - \exp\left(-\int_0^\tau \tilde{I}_o\, d\tau'\right)\right]\right\}$$

where $\tilde{I}_o = I_o/\bar{I}_o$. The number density is seen to be a hyperbolic tangent function. η is raised from η_0 ahead of the bleaching wave ($\xi \gg \xi_w(\tau)$) to 1 behind the wave ($\xi \ll \xi_w(\tau)$). When $\eta = 1$, the substance is said to be saturated.

The expression for $\xi_w(\tau)$ is composed of a steady-state term and a transient term. Considering only the steady-state term, the velocity of the bleaching wave is given by $V = 2I_o(t)/h\nu(n - 2n_{uo})$. The velocity of the bleaching wave can also be found by using the conservation of energy. This is done by requiring that the energy flux delivered by the laser (energy per unit time per unit area) must be equal to the energy required per unit volume to bleach the substance times the velocity of the bleaching wave. The calculation is made possible by the fact that essentially all of the laser energy is added to a thin region, the front of the bleaching wave. However, the conservation of energy cannot be used to calculate the velocity of the bleaching wave in a plasma. For this case the energy continues to be added to the plasma far behind the actual front of the bleaching wave. A plasma never saturates completely as does a two level substance since an infinite plasma temperature is required for perfect transparency.

The thickness of the bleaching wave is roughly the distance in which the argument of the hyperbolic tangent makes a change of one, so that $\Delta x_{wave} \sim 2/\sigma\nu(n - 2n_{uo})$ which is twice the initial absorption length. For the particular example of $\eta_o = 0$ (no molecules initially in the upper state) and a uniform laser pulse, $\tilde{I}_o = 1$, the solution becomes

$$\eta = \tfrac{1}{2}[1 + \tanh(\xi_w - \xi)]; \qquad \xi_w = \tfrac{1}{2}[\tau + \log(1 - e^{-\tau})]$$

and the bleaching wave velocity is $V = 2I_o/nh\nu$. This example is shown graphically in Fig. 2.

A comparison with the laser heated plasma in Fig. 1 shows a striking resemblance. While certain features differ between the two cases, a bleaching wave is seen to appear in both. Ahead of the wave there is apparently little radiation since the variable (η or T) changes slowly. In the front of the wave, the variable rises rapidly, due to strong irradiation and a short absorption length. Behind the wave, the variable rises much more slowly since the substance has been bleached and the absorption length has become very large.

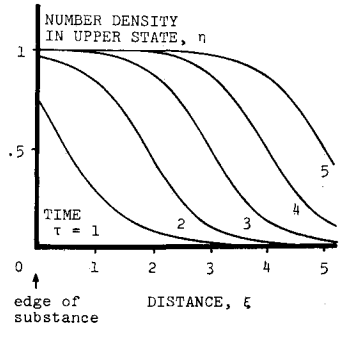

Fig. 2 Laser irradiation of a two level substance.

Laser Induced Breakdown Wave

For many conditions, an un-ionized gas such as air is essentially transparent to laser radiation. At least the absorption lengths are quite long compared to absorption lengths that arise in partially and fully ionized gases, or from resonant line absorption. Nevertheless, an extremely intense laser beam can cause breakdown of the gas. This phenomenon was first observed experimentally by Damon and Tomlinson[10] and by Meyerand and Haught.[11]

Studies of the mechanism of laser induced breakdown show that the breakdown begins with either stray free electrons in the gas, or with electrons produced by the multiphoton effect. The multiphoton effect is due to the simultaneous bombardment of an atom by several quanta of light, with enough total energy to free an electron. Once an electron is free, the large oscillating electric field causes the electron to collide with atoms and free more electrons. The cascading process produces a large number of electrons in a very short time. A clearly defined threshold intensity characterizes this breakdown. Several theoretical investigations of the breakdown phenomenon followed the experimental discovery.[12–16]

The dynamical behavior following the initial breakdown was seen to exhibit unusual effects. The expansion of the ionized region is not uniform in all directions. Rather, it moves much more rapidly in the cone of the focused laser beam. This phenomenon was first studied by Ramsden and Savic[17] who described it as a radiative detonation wave, that is a detonation wave whose fuel is the energy absorbed from the laser beam.

If the intensity of the laser beam appreciably exceeds the threshold intensity for breakdown at the focus, then another mechanism is possible. In this case the intensity will also exceed the threshold over a certain length of the beam and breakdown will occur at these points also. The gas in the breakdown region is stationary and the wavelike propagation of the region toward the laser source is due to the breakdown limit being reached at successively longer times for points closer to the source. The speed of this breakdown wave may exceed the speed of all of the possible gas dynamical processes. The breakdown wave was studied by Raizer[18] and its speed was calculated by a phenomenological model.

It is shown here that the breakdown is governed by the beam equation. The only variation is due to the non-one-dimensional geometry and the nonbleachable nature of the medium which cause the wave to move backwards, i.e. toward the laser source as contrasted to the usual sense of a bleaching wave moving away from the source. Otherwise one sees all the usual features of a bleaching wave. On one side of the wave (behind) is a nearly opaque plasma. On the other side (ahead) is a transparent un-ionized gas. The heating of the gas is done by absorption of the laser radiation—most of which occurs very rapidly at the front of the wave.

The nature of the breakdown wave is seen by considering a solution to a simple breakdown model. The radiative transfer equation is given in terms of the power, W,

$$\partial W/\partial x - W/\ell = 0 \tag{19}$$

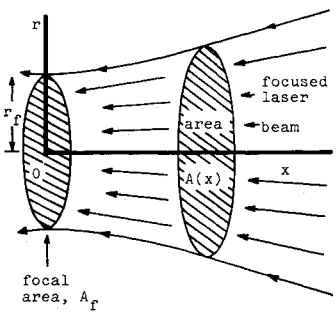

Fig. 3 Geometry of the focal region.

The opposite sign from the usual equation arises since the beam is assumed to travel in the $-x$ direction as shown in Fig. 3. Figure 3 also shows the geometry of the focal region. At any point the intensity is related to the power by $I = W/A(x)$, where $A(x)$ is the cross-sectional area of the beam.

The energy conservation equation is

$$\rho A(x) C_v(\partial T/\partial t) = \partial W/\partial x \tag{20}$$

where $\rho A(x)$ is analogous to the density in Eq. (1) and the power W is analogous to the intensity. Combining Eqs. (19) and (20) assuming both to be constant yields,

$$0 = \partial/\partial x\{[A(x)/k]\partial T/\partial t\} - A(x)\partial T/\partial t$$

which is clearly analogous to Eq. (4), the beam equation.

This problem can be solved with the following two assumptions. First the absorption length is given by

$$\ell = \ell_o/H(n_e - n_{et}) \tag{21}$$

where ℓ_o is a short absorption length corresponding to the gas after breakdown, n_{et} is the electron number density for breakdown, and $H(\ )$ is the Heaviside step function. Thus there is no absorption for electron densities below the threshold, and strong absorption above the threshold.

Following Raizer,[18] the second assumption is that the electron number density grows at a rate proportional to the intensity times the number density,

$$\partial n_e/\partial t = (A_f/t^* W_{\max}) n_e W/A(x) \tag{22}$$

where A_f is the focal area, $W_{\max}$ is the maximum power and t^* is the time of breakdown at the focal area.

Since the absorption length depends on the electron density, it is easier to solve the problem in terms of n_e and then find the temperature. Combining Eqs. (19), (21), and (22) to eliminate ℓ and W yields

$$(\partial^2/\partial t\,\partial x)[A(x)\log n_e] - (1/\ell_o)H(n_e - n_{et})\partial[A(x)\log n_e]/\partial t = 0 \tag{23}$$

The initial conditions are on the electron density and temperature, $n_e(x, O) = n_{eo}$, and $T(x, O) = T_o$, where n_{eo} is a small residual electron density due to chance free electrons in the gas, or due to electrons produced by the multiphoton effect.[18]

The primary boundary condition is $W(\infty, t) = W_o(t)$ but a more useful form can be written. Consider that for $n_e < n_{et}$ there is no absorption. Consider also that the successive points of breakdown (where $n_e = n_{et}$) will move in the $+x$ direction as time proceeds. If at some point in time, breakdown has not occurred at a point x, then it surely has not occurred at any place between x and the laser. Thus the intensity at x must be the undiminished intensity delivered by the laser so that the boundary condition can be rewritten as $W(x, t) = W_o(t)$, for $n_e < n_{et}$.

Equation (23) can be solved in two parts; for $n_e < n_{et}$ and for $n_e > n_{et}$, and these two parts must be matched. Considering first $n_e < n_{et}$, Eq. (23) can be integrated, and applying the initial condition and the boundary condition yields

$$\frac{n_e}{n_{eo}} = \exp\left[\frac{A_f}{t^* W_{\max}} \frac{1}{A(x)} \int_0^t W_o\, dt'\right] \tag{24}$$

which is valid for $n_e < n_{et}$, and where Eq. (22) was used in applying the boundary condition. Since the absorption length is infinite for $n_e < n_{et}$, the temperature will not change, but remains at T_o in this region.

The location of the breakdown wave $x_w(t)$ is at the point where $n_e = n_{et}$ which using Eq. (24) is given implicitly by

$$\frac{W_{\max} t^*}{A_f} \log \frac{n_{et}}{n_{eo}} = \frac{1}{A[x_w(t)]} \int_0^t W_o\, dt' \tag{25}$$

Of course this is precisely the point where the solution, Eq. (24) becomes invalid.

Now the solution for $n_e > n_{et}$ must be constructed. Then Eq. (23) becomes

$$(\partial/\partial t)\{\partial[A(x)\log n_e]/\partial x - A(x)/\ell_o \log n_e\} = 0$$

This equation can be integrated with respect to time, resulting in a first-order linear differential equation in x for $A(x)\log n_e$.

Solving this equation yields

$$\log n_e = a(t)[e^{x/\ell_o}/A(x)] + b(x) \tag{26}$$

At this point it is possible to go directly to an equation for the temperature behind the wave rather than solving for n_e. Using Eqs. (22) and (26), the laser power at a point behind the wave is

$$W = (t^* W_{\max}/A_f)a'(t)e^{x/\ell_o}$$

Applying the boundary condition to the front of the wave $W(x_w(t), t) = W_o(t)$ yields,

$$W = W_o(t)\exp\{[x - x_w(t)]/\ell_o\} \tag{27}$$

Then the temperature can be found by applying Eq. (27) to the energy conservation Eq. (20) and integrating with respect to time. Performing the integration and using $T(x_w(t), t) = T_o$ (temperature at the front is at its original value) to evaluate the constant of integration yields,

$$T = T_o + \frac{1}{\rho \ell_o C_v A(x)} \int_0^t H[x_w(t') - x] W_o(t') \exp\{[x - x_w(t')]/\ell_o\} dt' \tag{28}$$

The wave motion is clearly seen in Eq. (28) where the Heaviside function in the integral implies there is no heating until the wave arrives, at which time the temperature begins to rise.

The velocity of the breakdown wave can be found by taking the time derivative of Eq. (25),

$$V = A_f W_o(t)/W_{\max} t^* [dA/dx]_{x=x_w(t)} \log n_{et}/n_{eo}$$

Two simple examples are presented which clearly demonstrate the wavelike character and the effect of the absorption length ℓ_o. For both examples it is assumed that the laser intensity is constant after the pulse begins at $t = 0$; $W = W_o H(t)$. The initial gas temperature is $T_o = 0$, and the geometry of the laser beam is conical down to the focus ($x = 0$); $A(x) = A_f(1 + \beta(x/r)_f)^2$, where β is the tangent of the cone half-angle. For the example shown in Fig. 4, the absorption length is assumed to be nonzero; $\beta\ell_o/r_f = \frac{1}{3}$. The example shown in Fig. 5 has $\ell_o = 0$. In Fig. 4 the temperature of the gas continues to rise after passage of the wave due to the nonzero absorption length of the broken down gas. Hence it is similar to the plasma heating wave shown in Fig. 1 except that the laser beam is coming from the other direction. The temperature behind the wave in Fig. 5 is independent of time which is similar to the irradiation of a two level substance shown in Fig. 2.

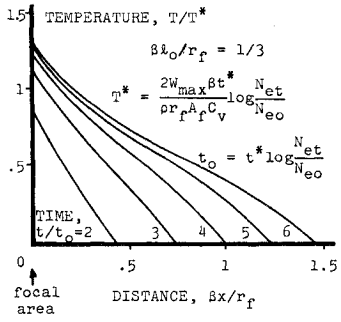

Fig. 4 The breakdown wave for nonzero absorption length in the plasma.

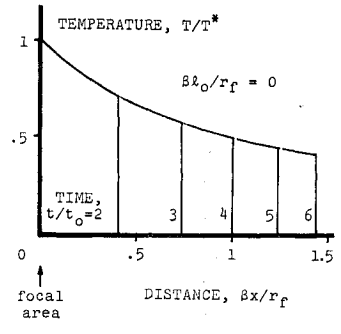

Fig. 5 The breakdown wave for zero absorption length in the plasma.

Actually in a plasma, $\ell = f(n_e, T)$ so for most cases the solution would be more complicated. However if the electron density in the wave front is greater than the critical electron density, the absorption length will become very short and Eq. (9) is not valid. In this case the energy is partially absorbed and partially reflected at the wave front. In this case Fig. 5 would more closely represent the situation.

The major difference between the breakdown wave and the bleaching waves is that it moves backwards and the propagation occurs due to the focused geometry of the beam, and the tendency of the air to become opaque rather than more transparent as it breaks down.

Thus, three coherent transport problems demonstrating wavelike character have been presented which bear similar characteristics. In the first two, laser irradiation of a layer of substance in time produced a significant increase in the absorption length. The result was a boring effect of the laser beam into the substance. In the third example, laser irradiation produced in time a sharp increase in the absorption coefficient. This, together with the focused geometry caused a reversed boring effect to appear. The penetration of the beam diminished with time. All three of these phenomena show similar profiles in the dependent variable, as seen in Figs. 1, 2, 4, and 5.

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